The Difficulty of Easy Projects*

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Abstract

We consider binary private contributions to public good projects that succeed when the number of contributors exceeds a threshold. We show that for standard distributions of contribution costs, valuable threshold public good projects are *more* likely to succeed when they require *more contributors*. Raising the success threshold reduces free-riding incentives, and this strategic effect dominates the direct effect. Common intuition that easier projects are more likely to succeed only holds for cost distributions with right tails fatter than Cauchy. Our results suggest government grants can reduce the likelihood that valuable, threshold public good projects succeed.

Keywords: public goods, pivotality, free-riding, thick tails

JEL Classification: D71, D82, H4

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1 Introduction

Common sense suggests that a regime that collapses if at least 10% of its citizens revolt is more fragile than a regime that collapses if at least 20% of its citizens revolt. It is also natural to think that, all else equal, public good projects that require the contribution of a larger fraction of the population (e.g., referendums with higher passage thresholds) are always less likely to succeed. We show this intuition is wrong for high value threshold public goods with binary contributions: among public good projects with high and *identical* values, those that require *more* participation for success are also *more* likely to succeed. For example, weakening the regime, lowering the passage threshold, or providing government grants can paradoxically reduce the likelihood of success in such projects.

We consider the classical threshold public good framework of Palfrey and Rosenthal (1984, 1988) (Fudenberg and Tirole, 1991, 6.2), in which the public good is provided whenever a sufficient number of citizens contribute. Figure 1 represents the game. $N \geq 2$ citizens must simultaneously decide whether to contribute to a public good project. The payoff of the status quo is normalized to 0. A project succeeds if at least q citizens contribute, where $1 \leq q \leq N$. If the project succeeds, each citizen receives a payoff $u > 0$. If citizen i contributes, she pays a cost c_i. We assume that c_i is citizen i's private information, and that $c_i \sim_{iid} F$ with $F(0) \in (0, 1)$. We allow for c_i to be negative to capture citizens who contribute regardless of what others do and the success of the project, which allows us to rule out the equilibrium in which no one contributes. To ease exposition, we assume that $F(c)$ is differentiable on its support.

		outcome	
		$n \geq q$	$n < q$
citizen i	*contribute*	$u - c_i$	$-c_i$
	not contribute	u	0

Figure 1: Threshold public good game with success threshold q. A public good with value u is provided if the number of citizens who contribute, n, exceeds the threshold q.

This framework captures the essence of various public good problems and has been adopted extensively in the literature. For example, "if 10 percent of the registered voters sign a petition at the city hall, a tax limitation referendum is held," or "fifty people see an assault taking place. If one person calls the police, the victim is saved" (Palfrey and Rosenthal, 1984, p.172). Other examples include voting in committees (Taylor and Ward, 1982), passing the budget in legislature (Myatt and Wallace, 2008), legislators voting to raise their own pay (Ordeshook, 1986, ch.3), participation in meetings (Osborne et al., 2000), boycotts (Diermeier and Van Mieghem, 2008), tender offers (Palfrey and Rosenthal, 1988), vaccination (Sandler, 2015), and lobbying (Hansen, 1985; Marwell and

Oliver, 1991), as well as the ratification of international treaties (Saijo and Yamato, 1999), and joining an alliance in a world war (Goeree and Holt, 2005). The large experimental literature that uses this framework includes Dawes et al. (1986), Offerman et al. (1996), Cadsby and Maynes (1999), Spiller and Bolle (2017), and Palfrey et al. (2017).

Our main result compares analytically the likelihood of public good provision for different success thresholds when the value of public good u is sufficiently small or sufficiently large (Theorem 1). We prove that when the value of public good u is small, the likelihood that the project succeeds is decreasing in the threshold q. This is consistent with the common sense that easier projects are more likely to succeed. In sharp contrast, we prove that when the value of public good u is sufficiently large, the likelihood that the project succeeds is *increasing* in the threshold q for cost distributions with bounded support, or with asymptotically increasing hazard rate.[1] We provide simulations to glean insight about how the likelihood of success varies with the success threshold for intermediate public good values.

The direct effect of raising the success threshold q reduces the likelihood of success: for the same citizens' behavior, it is less likely that at least $q+1$ of them contribute than that at least q of them contribute. But there is also an indirect effect. A higher success threshold changes the probability that a citizen is pivotal, and hence her incentives to contribute. When the public good value u is large, a higher success threshold raises the probability of pivotality and hence contribution incentives. We characterize when this indirect effect dominates.

To see that the indirect effect can dominate, consider the special case with unanimity and bounded support for contribution costs: $N = 2$, $c_i \sim_{iid} U[-1, 1]$, and $u = 1$. For $q = 2$, there is a unique symmetric equilibrium in which both citizens contribute and the likelihood of success is 1. When the success threshold falls from $q = 2$ to $q = 1$ and citizens do not adjust their behavior, the probability of success remains 1. So the direct effect is null. But the probability of being pivotal decreases, so each citizen contributes less, thereby reducing the likelihood of success.

One may conjecture that the logic of unanimity is the key driver of the result. If it were so, moving from $q = N - 1$ to unanimity would always raise the likelihood of success for sufficiently high public good values. This conjecture is wrong. The simplicity of the example stems from the bounded support of contribution costs, not unanimity. Focusing on distributions with power-law right tails ($\Pr(c_i > x) = x^{-\alpha}$, with $\alpha > 0$, e.g., Pareto distribution), we prove that, when public good value is sufficiently large, the likelihood of success is increasing in q *if and only if* $\alpha > 1$ (Proposition 1). When $\alpha \leq 1$, the likelihood of success is *decreasing* in q, no matter how high the public good value is even when we raise q to unanimity. As we will show, the key, instead, is the rate at which $1 - F$ falls.

Our results generate counter-intuitive predictions in many settings. When legislators

[1] That is, when $1 - F(c)$ is log-concave for sufficiently large c. If $f(c)$ is log-concave, so is $1 - F(c)$.

are voting to grant themselves a pay raise (u), higher supermajority requirements (higher q) may lead to higher probability of a raise, especially when the raise is large relative to the public backlash from voting yes on this unpopular issue (c).[2] Strengthening a moderately unpopular regime (raising q when u is moderate) makes it more likely to survive, while strengthening a more unpopular regime (raising q when u is large) leads to its collapse.

Our results also speak to the effect of government grants on the likelihood of threshold public good provision. Government grants in our setting correspond to reducing the success threshold q. Our results suggest that government grants may decrease the likelihood that a public good project succeeds. This result contrasts with the effect of government grants in the literature on public goods and charitable donations, in which the crowding out effect of government grant never more than eliminates the value of the grant (Roberts, 1984; Bergstrom et al., 1986; Andreoni and Payne, 2003). In these models, for a fixed level of public good, a citizen's incentive to contribute weakly increases with the fraction of the good financed by the government. In contrast, in our model, the effect of government contributions on the relationship between the probability of success and citizen incentives (captured by the probability of pivotality) is more complex.[3]

Our focus is on how variations in the success threshold q affect the likelihood a public good project succeeds for a given group size N. In Section 3, we generalize our main result, e.g., showing how the likelihood of success depends on $N - q$ and the distribution of contribution costs when the public good value u is sufficiently large (Theorem 2). This result contributes to the long-standing question of the relationship between public good provision and group size, dating back to Olson (1965).

Our paper is also related to the literature on the underdog effect in elections, in which the turnout rate of the majority is lower than the minority due to higher free riding incentives (Levine and Palfrey, 2007; Krasa and Polborn, 2009; Taylor and Yildirim, 2010; Myatt, 2015). In these models, the threshold q is endogenously determined by voter turnout. They show that raising a candidate's popularity can be completely offset by free riding, but free riding does not dominate the direct effect of higher popularity, and the

[2] Spenkuch et al. (2018) and Shakiba (2019) provide evidence that U.S. Senators routinely behave as if it is costly for them to vote for their preferred outcome, and that Senators vote strategically, based on pivotality considerations.

[3] These differences stem from different modeling choices. The cited models consider continuous contribution and public good levels, but abstract from strategic uncertainties that arise from asymmetric information. We follow the literature with binary contribution and public good levels, but allow for asymmetric information. Investigating the combination of these features is left for future work. However, two points are noteworthy. (1) The intuition for our results suggests that they qualitatively extend when the feasible contribution set is sufficiently coarse. In many applications, there are effectively few possible contribution levels as social norms or other costs restrict contributions to a few salient levels, or the nature of contribution is discrete (e.g., volunteering time and skills for Habitat for Humanity). At the same time, asymmetric information seems to play an important role in those applications. (2) Similar results arise in the cited models if the warm glow payoff that citizens receive from their contributions is increasing in the contributions of other citizens, but not the government's—so that government contribution can lower the marginal benefit of contributions for each level of public good.

more popular candidate is always weakly more likely to win.[4]

2 Analysis

The strategy of citizen i is a mapping from her private information c_i to a decision whether to contribute. Fix a strategy profile of citizens other than i, σ_{-i}, and let $\Pr(piv|\sigma_{-i})$ be the probability that i is pivotal given σ_{-i}. Then citizen i with cost c_i contributes to the project if $c_i < \Pr(piv|\sigma_{-i})\, u$ and does not contribute if $c_i > \Pr(piv|\sigma_{-i})\, u$. Thus, the best response of player i is a threshold strategy. We focus on symmetric equilibria.[5] Since $F(0) \in (0,1)$, equilibrium threshold $c_q^*(u)$ is characterized by the indifference condition[6]

$$\frac{c_q^*(u)}{u} = \binom{N-1}{q-1} \left(F\left(c_q^*(u)\right)\right)^{q-1} \left(1 - F(c_q^*(u))\right)^{N-q} \equiv \Pr\left(piv|q, c_q^*(u)\right). \tag{1}$$

Let $S_q(c_q^*(u))$ be the probability of success corresponding to the equilibrium threshold $c_q^*(u)$. Then,

$$S_q(c_q^*(u)) = \sum_{k=q}^{N} \binom{N}{k} \left(F\left(c_q^*(u)\right)\right)^k \left(1 - F\left(c_q^*(u)\right)\right)^{N-k} \tag{2}$$

$$= 1 - \sum_{k=0}^{q-1} \binom{N}{k} \left(F\left(c_q^*(u)\right)\right)^k \left(1 - F\left(c_q^*(u)\right)\right)^{N-k}. \tag{3}$$

Our goal is to characterize how changes in q affect the equilibrium probability of success $S_q(c_q^*(u))$. For a general public good value, this analysis is intractable. We prove results for the polar opposite cases where the public good value is sufficiently small or sufficiently large.

Theorem 1. *If the support of costs is bounded from above, or if $1 - F(x)$ is log-concave for sufficiently large x, then the likelihood of success is increasing in the success threshold when the public good value is sufficiently large: $S_{q+1}(c_{q+1}^*(u)) > S_q(c_q^*(u))$ for sufficiently large u. In contrast, the likelihood of success is always decreasing in the success threshold when the public good value is sufficiently small: $S_{q+1}(c_{q+1}^*(u)) < S_q(c_q^*(u))$ for sufficiently small u.*

The result is driven by the interplay of two effects. When the threshold q required for success increases to $q+1$, the direct effect decreases the probability of success: fixing

[4] In Myatt (2015), a candidate who is more popular but less popular than expected will lose, but raising candidate's realized popularity improves her chances.

[5] For a defense of symmetric equilibria see Palfrey and Rosenthal (1984) and Dixit and Olson (2000).

[6] When the distribution of costs has bounded support, the threshold that satisfies the indifference condition may be above the upper bound of the support, in which case $F\left(c_q^*(u)\right) = 1$.

citizen behavior, it is less likely that at least $q+1$ of them contribute than that at least q of them contribute. However, there is an indirect effect. As q increases, the probability that citizen i is pivotal also changes, which affects her behavior.

When the value of public good u is large, each citizen contributes with large probability, so it is *more* likely that exactly q citizens other than i contribute than that exactly $q-1$ citizens other than i contribute. Thus, as q increases to $q+1$, citizen i becomes *more likely to be pivotal*, and hence she increases her contribution threshold. So the indirect effect counters the direct effect. For the indirect effect to dominate, it must be that as the equilibrium threshold increases, the expected number of new participants (those whose participation costs c_is are just above the old equilibrium threshold c_q^*) is sufficiently large. When $F(c)$ increases more quickly, it means there are more such citizens; and a bounded support or log-concavity of $1-F(c)$ means that $F(c)$ increases sufficiently fast.

In contrast, when the value of public good u is sufficiently small, even an increase in the probability of pivotality has a negligible effect on citizens' incentives to contribute. Therefore, the direct effect always dominates the indirect effect, so the likelihood of success decreases.[7]

Theorem 1 does not address whether our result for large public good values extends to distributions with thicker right tails for which $1-F(x)$ is not log-concave for large x. Proposition 1 provides the analysis for the class of distributions with power-law right tails (e.g., Pareto distribution).

Proposition 1. *Suppose $1-F(x)=\beta/x^{\alpha}$, $\alpha,\beta>0$, for sufficiently large x. When the public good value is sufficiently large, the likelihood of success is increasing in the success threshold, $S_{q+1}(c_{q+1}^*(u))>S_q(c_q^*(u))$, if and only if $\alpha>1$.*

Three points are worth highlighting. First, combining Theorem 1 and Proposition 1 suggests that $1/x$ is close to the boundary tail distribution that separates cost distributions for which the strategic effect dominates (thinner than Cauchy) from those for which the direct effect dominates (thicker than Cauchy)—recall that for the standard Cauchy distribution, $\Pr(c_i>x)\approx 1/\pi x$ for large x.

Second, based on our simple example in the Introduction, one may have thought that unanimity is the key for a higher success threshold to increase the chances of success when u is sufficiently large. However, unanimity is neither necessary nor sufficient for this result. It is not sufficient because raising the success threshold from $q=N-1$ to $q+1=N$ *reduces* the likelihood of success when $1-F(x)=x^{-\alpha}$ for $\alpha\leq 1$, no matter how large u is. It is not necessary because raising the success threshold at any level (e.g.,

[7] When u is small, each citizen contributes with probability close to $F(0)$. When $F(0)$ is small, the probability of being pivotal decreases with q: if each citizen contributes with low probability, it is *less* likely that exactly q citizens other than i contribute than that exactly $q-1$ citizens other than i contribute. In this case, the indirect effect strengthens the direct effect.

from $q = N-3$ to $q+1 = N-2$) increases the likelihood of success when $1-F(x)$ goes to 0 faster and u is sufficiently large.[8]

Third, a higher success threshold increases the chances of success only if it raises the contribution threshold: $S_{q+1}(c^*_{q+1}) > S_q(c^*_q)$ requires $c^*_{q+1} > c^*_q$. This may lead one to conjecture that a very large c^*_{q+1}/c^*_q is sufficient for $S_{q+1}(c^*_{q+1}) > S_q(c^*_q)$. This conjecture is incorrect. To see this, suppose $1-F(x) = x^{-\alpha}$ with $\alpha > 0$ for large x, and compare $c^*_N(u)$ and $c^*_1(u)$ when u is very large. The indifference conditions imply

$$\lim_{u\to\infty} \frac{c^*_N(u)}{u} = 1 \text{ and } \lim_{u\to\infty} \frac{c^*_1(u)}{u^{\frac{1}{1+\alpha(N-1)}}} = 1.$$

Thus, $c^*_N(u)/c^*_1(u)$ can be made arbitrary large by choosing a sufficiently large u. However, Proposition 1 shows that, no matter how large u becomes, $S_N(c^*_N(u)) < S_1(c^*_1(u))$ for $\alpha \in (0,1]$. In fact, when $q \in \{1,\cdots,N-2\}$, for $S_{q+1}(c^*_{q+1}(u)) > S_q(c^*_q(u))$ to hold for sufficiently large u, c^*_{q+1}/c^*_q must *not* be too large.[9] To see this, consider equations (1) and (3) and suppose u is very large. A citizen almost always contributes, so that the dominant term in these equations is $1-F(c^*_q(u))$ with the lowest power

$$Pr(piv|q, c^*_q) \approx \binom{N-1}{q-1}\left(1-F(c^*_q)\right)^{N-q} \quad \text{and} \quad S_q(c^*_q) \approx 1 - \binom{N}{q-1}\left(1-F\left(c^*_q\right)\right)^{N-q+1}.$$

For $S_{q+1}(c^*_{q+1}) > S_q(c^*_q)$, we want $\left(1-F\left(c^*_{q+1}\right)\right)^{N-q}$ to be as small as possible. But this does *not* mean that we want c^*_{q+1} to be as large as possible, because c^*_{q+1} is an equilibrium object and is linked to $\left(1-F\left(c^*_{q+1}\right)\right)^{N-q}$ through the likelihood of pivotality: in equilibrium, if c^*_{q+1} is higher, then so is $1-F(c^*_{q+1})$. Rather, as Theorem 1 and Proposition 1 show, the key is how fast $1-F$ falls. When raising q to $q+1$, if the equilibrium contribution threshold did not change, we would have

$$\binom{N-1}{q}\left(1-F(c^*_q)\right)^{N-(q+1)} > \binom{N-1}{q-1}\left(1-F(c^*_q)\right)^{N-q} \approx \frac{c^*_q}{u}.$$

But now the equilibrium contribution threshold increases to c^*_{q+1} to restore indifference:

$$\binom{N-1}{q}\left(1-F(c^*_{q+1})\right)^{N-(q+1)} \approx \frac{c^*_{q+1}}{u}.$$

It follows that the faster $1-F(\cdot)$ drops around c^*_q, the lower will be $1-F(c^*_{q+1})$, and hence the higher will be $S_{q+1}(c^*_{q+1})$.

Our analytical results focus on the asymptotic cases when the public good value is very

[8] Moreover, note that for all $q \in \{1,\cdots,N-1\}$, unlike for unanimity, each player is unlikely to be pivotal even when u is very large, while each player contributes with probability close to 1.

[9] See part (iii) of Lemma 1 in the Appendix.

small or very large. How does the likelihood of success vary with the success threshold for public good values away from the extremes? To glean insights, we provide simulations. First, consider the example in the Introduction, but with Normal and Cauchy distributions. Figure 2 illustrates. With $F = N(1,1)$, raising the success threshold from 1 to 2 increases the likelihood of success whenever the public good value u is above a threshold of about 2.52. Around this public good value, for $q = 2$, a citizen participates and is pivotal with probability 0.90. In contrast, with $F = Cauchy(0,1)$, raising the success threshold always reduces the likelihood of success, e.g., even when $u = 100$ at which a citizen participates with probability 0.9968 and is pivotal with probability 0.997 under unanimity—for $q = 1$, each citizen is pivotal with probability 0.056. Next, suppose $N = 3$ and raise the success threshold from $q = 1$ to $q = 2$. For $F = N(2,1)$, when the public good value is higher than 8.11, again, the likelihood of success falls. Around this public good value, for $q = 2$, a citizen participates with probability 0.78 and is pivotal with probability 0.34. For $F = Cauchy(0,1)$, at $u = 8.1$, the probability of pivotality is 0.25 when $q = 2$ and 0.92 when $q = 3$, and the probability of success always decreases in q.[10]

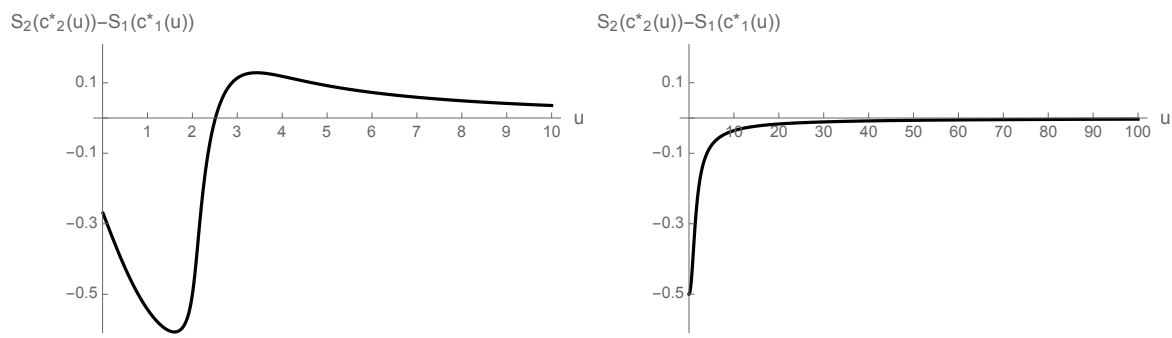

Figure 2: The difference between success probability when $q = 2$ and when $q = 1$ for a game with two players when the distribution of costs is $N(1,1)$ (left figure) and Cauchy with location 0 and scale 1 (right figure).

These simulations further confirm that unanimity, a high probability of pivotality, or a very high probability of participation are not the driving force behind our results. They do indicate, however, that the public good value has to be large relative to individual contribution costs. Indeed, in many applications, the public good value is far larger than a typical individual's contribution costs. For example, using the data from Texas local liquor referendums, Coate et al. (2008) estimate the average value of passage to potential voters as 35 times larger than the average voting costs. In the applications mentioned in the Introduction (e.g., reporting a crime (Palfrey and Rosenthal, 1984) or boycotting a product (Diermeier and Van Mieghem, 2008)), one expects that the public good value is large enough relative to individual participation costs, so that any player would be willing to pay her participation costs if she were the only player and her participation would

[10]There is a unique symmetric equilibrium for the parameters of our simulations.

deliver the good.

3 Generalized Theorem

Our intuition suggests a close link between the effect of raising the success threshold q and reducing the group size N. We now extend Theorem 1, showing how the likelihood of success depends on $N - q$. We state the generalized theorem below, and provide its proof and the analogous generalization of Proposition 1 in an Online Appendix. Let $c^*_{q,N}(u)$ be the symmetric equilibrium threshold in the game with success threshold q and N players.

Theorem 2. *Take q, N, q' and N' such that $q \leq N$ and $q' \leq N'$.*

1. *If $q \leq q'$ and $N - q \geq N' - q'$, with at least one inequality strict, then $S_{q,N}(c^*_{q,N}(u)) > S_{q',N'}(c^*_{q',N'}(u))$ for sufficiently small u.*

2. *Suppose the support of costs is bounded from above, or $1 - F(x)$ is log-concave for sufficiently large x. If $N - q < N' - q'$, then $S_{q,N}(c^*_{q,N}(u)) > S_{q',N'}(c^*_{q',N'}(u))$ for sufficiently large u.*

Theorem 2 sheds light on the long-standing question of the relationship between group size and public good provision. Olson (1965) argued that larger groups are less likely to provide public goods because they have a more severe free riding problem. However, Chamberlin (1974) (and many since) showed that Olson's conjecture does not hold in standard public good models like Bergstrom et al. (1986).[11] The direct effect of raising group size dominates the strategic, free riding effect, so that larger groups provide more public goods. In symmetric mixed strategy equilibria of the complete information threshold public good game with $q = 1$ (analyzed in Palfrey and Rosenthal (1984)), Diekmann (1985) showed that the probability of success is decreasing in group size N. We show this result is not robust to the strategic uncertainty introduced by private participation costs.[12] When the public good value is small, the likelihood of public good provision is increasing in N. In contrast, for sufficiently valuable public goods, the strategic effect dominates and Olson's conjecture holds for any success threshold $q \leq N$ if either the distribution of contribution costs are bounded from above or have log-concave right tails. Our generalization of Proposition 1 (in the Online Appendix) shows that when the right

[11] In Esteban and Ray (2001), multiple groups compete for a prize. When the prize is a purely public good, the probability of success is increasing in group size.

[12] In particular, consider $c_i = c + \sigma\epsilon_i$. For any σ, Diekmann's result is overturned for sufficiently small u if $F(c_i)$ satisfies our conditions, and for sufficiently large u if the distribution of ϵ_i has fat tails. In a dynamic extension of Palfrey and Rosenthal (1984, 1988), Yildirim (2006) finds that in later periods when dynamic incentives are absent, participation is decreasing in group size when participation costs have bounded support.

tail distribution of participation costs is fatter than Cauchy, Olson's conjecture is reversed even for very high value public goods.[13]

4 Conclusion

Public good projects that require fewer contributors to succeed may seem to have a higher chance of success. That fewer contributors are needed for success may increase free riding incentives, but common sense suggests that this strategic effect should not offset the direct effect. We show that in a standard threshold public good game with binary contributions, this intuition fails under quite standard conditions.

Several directions for future research stand out. (1) We focused on threshold public good with binary contributions. The analogous analysis with multiple contribution levels is left to future research. (2) We analyzed the standard case in which costs are uncorrelated. Shadmehr (2019) provides a partial characterization of the same game in the limit when the number of citizens is large and costs have common and idiosyncratic components. With a numerical example, he shows that with Normal prior about the common cost component, the likelihood of success can be increasing in the success threshold. Analyzing the problem for intermediate correlation levels remains to be done. (3) The simple setting makes testing predictions in lab experiments feasible and especially valuable given the counter-intuitive nature of the results.

[13] Deserranno et al. (2020) look at a random entry of NGOs into areas of Uganda that already have government-provided healthcare workers. The entry by NGOs reduces the probability of receiving health care from any healthcare worker (as measured by whether a household "received medical care" in the past year) by twelve percentage points. This finding is consistent with part 2 of Theorem 2 in that increasing the number of potential healthcare providers leads to a lower probability of healthcare provision.

Appendix: Proofs

To ease the exposition, we drop superscript * in $c_q^*(u)$, and write $c_q(u)$ or c_q for equilibrium thresholds, recognizing that c_q depends on model parameters, including u. We also use $S_q(u)$ for $S_q(c_q^*(u))$.

We begin with a preliminary lemma that will be used in the proof of Theorem 1. Part (iii) of this lemma bounds $\frac{c_{q+1}}{c_q}$ as $u \to \infty$. As discussed in the main text, ensuring that $\frac{c_{q+1}}{c_q}$ is *not* too large is crucial for the argument.

Lemma 1. *(i)* $\lim_{u\to\infty} \frac{c_N(u)}{u} = 1$. *Moreover, if the support of costs is bounded from above, or if* $1-F(x)$ *is log-concave for sufficiently large* x*, then (ii) for all* $\alpha > 0$, $\lim_{u\to\infty} \frac{c_q(u)}{u^\alpha} = 0$ *if* $q < N$*, and (iii)* $\lim_{u\to\infty} \frac{c_{q+1}(u)}{c_q(u)} < \infty$ *if* $q < N-1$.

Proof of Lemma 1. Part (i) directly follows from (1).

To prove parts (ii) and (iii), suppose first the support of costs is bounded from above. Then for $q < N$, $c_q(u)$ is strictly smaller than the upper bound of the cost distribution, so it is bounded from above (which proves part (ii)), and bounded away from 0 (which proves part (iii)).

Next, suppose the support of costs does not have an upper bound, but $1 - F(x)$ is log-concave for sufficiently large x.

Part (ii). Differentiating (1) with respect to u yields

$$\frac{dc_q(u)}{du} = \frac{\binom{N-1}{q-1}\left(F\left(c_q\right)\right)^{q-1}\left(1-F\left(c_q\right)\right)^{N-q}}{1-u\binom{N-1}{q-1}\left(F\left(c_q\right)\right)^{q-2}\left(1-F\left(c_q\right)\right)^{N-q-1}\left(\left(q-1\right)\left(1-F\left(c_q\right)\right)-(N-q)F(c_q)\right)f\left(c_q\right)}. \tag{4}$$

From (1), the numerator is equal to $\frac{c_q}{u}$, and also

$$\binom{N-1}{q-1}\left(F\left(c_q\right)\right)^{q-2}\left(1-F\left(c_q\right)\right)^{N-q-1} = \frac{c_q(u)}{u}\frac{1}{F(c_q)\left(1-F(c_q)\right)}. \tag{5}$$

Substituting $\frac{c_q}{u}$ and (5) into (4) yields:

$$\begin{aligned}\frac{dc_q(u)}{du} &= \frac{\frac{c_q}{u}}{1-u\frac{c_q}{u}\frac{1}{F(c_q)(1-F(c_q))}\left(\left(q-1\right)\left(1-F\left(c_q\right)\right)-(N-q)F(c_q)\right)f\left(c_q\right)} \\ &= \frac{1}{u}\frac{1}{\frac{1}{c_q}-\left((q-1)\frac{1-F(c_q)}{F(c_q)}-(N-q)\right)\frac{f(c_q)}{1-F(c_q)}}.\end{aligned} \tag{6}$$

From (1), $\lim_{u\to\infty} c_q(u) = \infty$, and hence $\lim_{u\to\infty} F(c_q(u)) = 1$. Using L'Hôpital's rule, we

have

$$\begin{aligned}
\lim_{u\to\infty}\frac{c_q(u)}{u^\alpha} &= \lim_{u\to\infty}\frac{\frac{dc_q(u)}{du}}{\alpha u^{\alpha-1}} \\
&= \lim_{u\to\infty}\frac{1}{u}\frac{1}{\frac{1}{c_q}-\left((q-1)\frac{1-F(c_q)}{F(c_q)}-(N-q)\right)\frac{f(c_q)}{1-F(c_q)}}\frac{1}{\alpha u^{\alpha-1}} \quad \text{(from (6))} \\
&= \lim_{u\to\infty}\frac{1}{\alpha}\frac{1}{1-\left((q-1)\frac{1-F(c_q)}{F(c_q)}-(N-q)\right)\frac{f(c_q)}{1-F(c_q)}\,c_q}\frac{c_q}{u^\alpha}.
\end{aligned}$$

Since $1-F$ is log-concave, $\lim_{u\to\infty}\frac{f(c_q(u))}{1-F(c_q(u))}>0$, and the second line above implies that $\lim_{u\to\infty}\frac{c_q(u)}{u^\alpha}<\infty$, so we can move $\lim_{u\to\infty}\frac{c_q(u)}{u^\alpha}$ to one side, obtaining

$$\lim_{u\to\infty}\frac{c_q(u)}{u^\alpha}\left(1-\frac{1}{\alpha}\frac{1}{1-\left((q-1)\frac{1-F(c_q)}{F(c_q)}-(N-q)\right)\frac{f(c_q)}{1-F(c_q)}\,c_q}\right)=0.$$

Because $\lim_{u\to\infty}\frac{f(c_q)}{1-F(c_q)}>0$, the limit of the term in the parenthesis is not 0 for any α, and hence $\lim_{u\to\infty}\frac{c_q(u)}{u^\alpha}=0$.

Part (iii). By contradiction, suppose that $\lim_{u\to\infty}\frac{c_{q+1}(u)}{c_q(u)}=\infty$. Then, $c_{q+1}(u)>c_q(u)$ for sufficiently large u. Because $1-F$ is log-concave, $\frac{f}{1-F}$ is increasing, and hence

$$\lim_{u\to\infty}\frac{\frac{f(c_q(u))}{1-F(c_q(u))}}{\frac{f(c_{q+1}(u))}{1-F(c_{q+1}(u))}}<\infty. \tag{7}$$

Moreover, using L'Hôpital's rule, we obtain

$$\begin{aligned}
\lim_{u\to\infty}\frac{c_{q+1}(u)}{c_q(u)} &= \lim_{u\to\infty}\frac{\frac{dc_{q+1}(u)}{du}}{\frac{dc_q(u)}{du}} \\
&= \lim_{u\to\infty}\frac{\frac{1}{c_q}-\left((q-1)\frac{1-F(c_q)}{F(c_q)}-(N-q)\right)\frac{f(c_q)}{1-F(c_q)}}{\frac{1}{c_{q+1}}-\left(q\frac{1-F(c_{q+1})}{F(c_{q+1})}-(N-q-1)\right)\frac{f(c_{q+1})}{1-F(c_{q+1})}} \quad \text{(from (6))} \\
&= \lim_{u\to\infty}\frac{N-q}{N-q-1}\frac{\frac{f(c_q(u))}{1-F(c_q(u))}}{\frac{f(c_{q+1}(u))}{1-F(c_{q+1}(u))}}<\infty \quad \text{(from (7))},
\end{aligned}$$

which is a contradiction. □

We now provide the proof of Theorem 1.

Proof of Theorem 1. First, we prove the second part of the theorem pertaining to low

u. From (1), $c_q(0) = c_{q+1}(0) = 0$. Thus, from (2),

$$\begin{aligned} S_{q+1}(0) - S_q(0) &= \sum_{k=q+1}^{N} \binom{N}{k} (F(0))^k (1-F(0))^{N-k} - \sum_{k=q}^{N} \binom{N}{k} (F(0))^k (1-F(0))^{N-k} \\ &= -\binom{N}{q} (F(0))^q (1-F(0))^{N-q} < 0. \end{aligned}$$

The result follows from the continuity of $c_q(u)$ and $S_q(u)$ in u.

Now, we prove the first part of the theorem. The difference between success probability for $q+1$ and q is as follows:

$$S_{q+1}(u) - S_q(u) = (1 - S_q(u)) \left(1 - \frac{1 - S_{q+1}(u)}{1 - S_q(u)}\right).$$

Thus, it suffices to show

$$\lim_{u\to\infty} \frac{1 - S_{q+1}(u)}{1 - S_q(u)} < 1.$$

From (3),

$$\begin{aligned} \frac{1 - S_{q+1}(u)}{1 - S_q(u)} &= \frac{\sum_{k=0}^{q} \binom{N}{k} (F(c_{q+1}))^k (1 - F(c_{q+1}))^{N-k}}{\sum_{k=0}^{q-1} \binom{N}{k} (F(c_q))^k (1 - F(c_q))^{N-k}} \\ &= \frac{\sum_{k=0}^{q} \binom{N}{k} (F(c_{q+1}))^k (1 - F(c_{q+1}))^{q-k}}{\sum_{k=0}^{q-1} \binom{N}{k} (F(c_q))^k (1 - F(c_q))^{q-1-k}} \frac{(1 - F(c_{q+1}))^{N-q}}{(1 - F(c_q))^{N-q+1}} \end{aligned} \tag{8}$$

Moreover, since $\lim_{u\to\infty} F(c_q) = \lim_{u\to\infty} F(c_{q+1}) = 1$,

$$\lim_{u\to\infty} \sum_{k=0}^{q} \binom{N}{k} (F(c_{q+1}))^k (1 - F(c_{q+1}))^{q-k} = \binom{N}{q}, \tag{9}$$

$$\lim_{u\to\infty} \sum_{k=0}^{q-1} \binom{N}{k} (F(c_q))^k (1 - F(c_q))^{q-1-k} = \binom{N}{q-1}. \tag{10}$$

Substituting (9) and (10) into (8) yields

$$\lim_{u\to\infty} \frac{1 - S_{q+1}(u)}{1 - S_q(u)} = \lim_{u\to\infty} \frac{\binom{N}{q}}{\binom{N}{q-1}} \frac{(1 - F(c_{q+1}))^{N-q}}{(1 - F(c_q))^{N-q+1}}. \tag{11}$$

For the rest of the proof, we consider two cases separately: $q < N-1$ and $q = N-1$. The latter case corresponds to moving to unanimity, and part (iii) of Lemma 1 does not apply for this case. Therefore, it requires a different approach.

$\mathbf{q < N-1}$. From the indifference condition (1),

$$(1-F(c_q))^{N-q+1} = \left(\frac{c_q}{u}\frac{1}{\binom{N-1}{q-1}(F(c_q))^{q-1}}\right)^{\frac{N-q+1}{N-q}}, \text{ for } q \in \{1,\cdots,N-1\},$$

$$(1-F(c_{q+1}))^{N-q} = \left(\frac{c_{q+1}}{u}\frac{1}{\binom{N-1}{q}(F(c_{q+1}))^{q}}\right)^{\frac{N-q}{N-q-1}}, \text{ for } q \in \{1,\cdots,N-2\}.$$

Substituting these into (11) yields that, for $q \in \{1,\cdots,N-2\}$,

$$\begin{aligned}
\lim_{u\to\infty}\frac{1-S_{q+1}(u)}{1-S_q(u)} &= \lim_{u\to\infty}\frac{N-q+1}{q}\frac{\left(\frac{c_{q+1}(u)}{u}\frac{1}{\binom{N-1}{q}(F(c_{q+1}(u)))^{q}}\right)^{\frac{N-q}{N-q-1}}}{\left(\frac{c_q(u)}{u}\frac{1}{\binom{N-1}{q-1}(F(c_q(u)))^{q-1}}\right)^{\frac{N-q+1}{N-q}}} \\
&= \lim_{u\to\infty}\frac{N-q+1}{q}\frac{\left(\frac{1}{\binom{N-1}{q}(F(c_{q+1}(u)))^{q}}\right)^{\frac{N-q}{N-q-1}}}{\left(\frac{1}{\binom{N-1}{q-1}(F(c_q(u)))^{q-1}}\right)^{\frac{N-q+1}{N-q}}}\left(\frac{c_{q+1}(u)}{c_q(u)}\right)^{\frac{N-q+1}{N-q}}\left(\frac{c_{q+1}(u)}{u}\right)^{\frac{1}{(N-q-1)(N-q)}} \\
&= \frac{N-q+1}{q}\frac{\left(\frac{1}{\binom{N-1}{q}}\right)^{\frac{N-q}{N-q-1}}}{\left(\frac{1}{\binom{N-1}{q-1}}\right)^{\frac{N-q+1}{N-q}}}\left(\lim_{u\to\infty}\frac{c_{q+1}(u)}{c_q(u)}\right)^{\frac{N-q+1}{N-q}}\left(\lim_{u\to\infty}\frac{c_{q+1}(u)}{u}\right)^{\frac{1}{(N-q-1)(N-q)}}.
\end{aligned}$$

From part (ii) of Lemma 1, $\lim_{u\to\infty}\frac{c_{q+1}(u)}{u} = 0$. From part (iii) of Lemma 1, $\lim_{u\to\infty}\frac{c_{q+1}(u)}{c_q(u)} < \infty$. Thus, $\lim_{u\to\infty}\frac{1-S_{q+1}(u)}{1-S_q(u)} = 0$, and the result follows.

$\mathbf{q = N-1}$. From (11), it suffices to show

$$\lim_{u\to\infty}\frac{1-S_N(u)}{1-S_{N-1}(u)} = \lim_{u\to\infty}\frac{2}{N-1}\frac{1-F(c_N)}{(1-F(c_{N-1}))^2} < 1. \tag{12}$$

We show $\lim_{u\to\infty}\frac{1-F(c_N)}{(1-F(c_{N-1}))^2} = 0$. If the support of costs is bounded from above, then $1-F(c_N(u)) = 0$ for sufficiently large u, while $1-F(c_{N-1}(u)) > 0$ for all u although it becomes arbitrarily close to 0. Next, consider the case where the support

of costs has no upper bound. From parts (i) and (ii) of Lemma 1, it suffices to show:

$$\lim_{u\to\infty} \frac{1-F(u)}{(1-F(u^\alpha))^2} = 0, \text{ for some } \alpha \in (0,1). \tag{13}$$

Because $1-F(x) \in (0,1)$ is decreasing and log-concave, its right-tail is at most exponential (An, 1998). In particular, for sufficiently large x, $1-F(x)$ goes to 0 faster than x^{-b}, for any $b > 0$.

Pick an $\alpha \in (0, 1/2)$. Fix $\epsilon > 0$. Pick a sufficiently large $\hat{x}$, such that for $x > \hat{x}$, $1-F(x)$ goes to 0 faster than x^{-b}, for any $b > 0$. Pick $x_1 > \hat{x}$, and find b_1 such that $1-F(x_1^\alpha) = (x_1^\alpha)^{-b_1} = x_1^{-\alpha b_1}$. Hence, $1-F(x_1) < x_1^{-b_1}$, and hence

$$\frac{1-F(x_1)}{(1-F(x_1^\alpha))^2} = \frac{1-F(x_1)}{x_1^{-2\alpha b_1}} < \frac{x_1^{-b_1}}{x_1^{-2\alpha b_1}} = \frac{1}{x_1^{b_1(1-2\alpha)}}.$$

If $\frac{1}{x_1^{b_1(1-2\alpha)}} < \epsilon$, then set $\bar{x} = \epsilon^{-\frac{1}{(1-2\alpha)b_1}}$. Otherwise, choose $\bar{x} > x_1$ large enough, so that $\frac{1}{\bar{x}^{b_1(1-2\alpha)}} < \epsilon$. Because $1-F(x)$ goes to 0 faster that x^{-b_1}, there exists a $b_2 > b_1$ such that $1-F(\bar{x}^\alpha) = (\bar{x}^\alpha)^{-b_2} = \bar{x}^{-\alpha b_2}$. Thus,

$$\frac{1-F(\bar{x})}{(1-F(\bar{x}^\alpha))^2} = \frac{1-F(\bar{x})}{\bar{x}^{-2\alpha b_2}} < \frac{\bar{x}^{-b_2}}{\bar{x}^{-2\alpha b_2}} = \frac{1}{\bar{x}^{b_2(1-2\alpha)}} < \frac{1}{\bar{x}^{b_1(1-2\alpha)}} < \epsilon.$$

Now, consider $x > \bar{x}$. Find b_3, so that $1-F(x^\alpha) = x^{-\alpha b_3}$. Again, because $1-F(x)$ goes to 0 faster than x^{-b}, for all $b > 0$, $b_3 > b_2$, and hence $\frac{1-F(x)}{(1-F(x^\alpha))^2} < \epsilon$. The result follows.

We have shown that for any given $q \in \{1, \cdots, N-1\}$, $S_{q+1}(u) > S_q(u)$ for sufficiently large u. Because there is a finite number of q's, $S_q(u)$ increases in q for sufficiently large u. □

Proof of Proposition 1. Substituting $1-F(x) = \frac{\beta}{x^\alpha}$ in (11) yields:

$$\lim_{u\to\infty} \frac{1-S_{q+1}(u)}{1-S_q(u)} = \lim_{u\to\infty} \frac{N-q+1}{\beta q} \left(\frac{(c_q(u))^{N-q+1}}{(c_{q+1}(u))^{N-q}} \right)^\alpha. \tag{14}$$

From indifference condition (1),

$$c_q(u) = \binom{N-1}{q-1} \left(1 - \frac{\beta}{(c_q(u))^\alpha}\right)^{q-1} \left(\frac{\beta}{(c_q(u))^\alpha}\right)^{N-q} u.$$

Thus,

$$(c_q(u))^{N-q+1} = \left(\binom{N-1}{q-1} \beta^{N-q} \left(1 - \frac{\beta}{(c_q(u))^\alpha}\right)^{q-1} u \right)^{\frac{N-q+1}{\alpha(N-q)+1}}.$$

Substituting this in (14) yields:

$$\begin{aligned}
\lim_{u\to\infty} \frac{1-S_{q+1}(u)}{1-S_q(u)} &= \lim_{u\to\infty} \frac{N-q+1}{\beta q} \left(\frac{\left(\binom{N-1}{q-1} \beta^{N-q} \left(1 - \frac{\beta}{(c_q(u))^\alpha}\right)^{q-1} u \right)^{\frac{N-q+1}{\alpha(N-q)+1}}}{\left(\binom{N-1}{q} \beta^{N-q-1} \left(1 - \frac{\beta}{(c_{q+1}(u))^\alpha}\right)^{q} u \right)^{\frac{N-q}{\alpha(N-q-1)+1}}} \right)^\alpha \\
&= \lim_{u\to\infty} \frac{N-q+1}{\beta q} \left(\frac{\left(\binom{N-1}{q-1} \beta^{N-q} u \right)^{\frac{N-q+1}{\alpha(N-q)+1}}}{\left(\binom{N-1}{q} \beta^{N-q-1} u \right)^{\frac{N-q}{\alpha(N-q-1)+1}}} \right)^\alpha \\
&= \lim_{u\to\infty} \frac{N-q+1}{\beta q} \left(\frac{\left(\binom{N-1}{q-1} \beta^{N-q} \right)^{\frac{N-q+1}{\alpha(N-q)+1}}}{\left(\binom{N-1}{q} \beta^{N-q-1} \right)^{\frac{N-q}{\alpha(N-q-1)+1}}} \right)^\alpha u^{\left(\frac{N-q+1}{\alpha(N-q)+1} - \frac{N-q}{\alpha(N-q-1)+1}\right)\alpha}.
\end{aligned}$$

Write the power of u as $\frac{\alpha}{(\alpha(N-q)+1)(\alpha(N-q-1)+1)}(1-\alpha)$, and recognize that $\frac{\alpha}{(\alpha(N-q)+1)(\alpha(N-q-1)+1)} > 0$. Thus,

$$\lim_{u\to\infty} \frac{1-S_{q+1}(u)}{1-S_q(u)} = \begin{cases} 0 & ;\alpha > 1 \\ \infty & ;\alpha < 1 \end{cases}, \quad \text{and hence} \quad \lim_{u\to\infty} \left(1 - \frac{1-S_{q+1}(u)}{1-S_q(u)}\right) = \begin{cases} > 0 & ;\alpha > 1 \\ < 0 & ;\alpha < 1 \end{cases}.$$

When $\alpha = 1$,

$$\lim_{u\to\infty} \frac{1-S_{q+1}(u)}{1-S_q(u)} = \frac{N-q+1}{\beta q} \frac{\binom{N-1}{q-1}\beta^{N-q}}{\binom{N-1}{q}\beta^{N-q-1}} = \frac{N-q+1}{N-q}.$$

Thus, for $\alpha = 1$, $\lim_{u\to\infty} \left(1 - \frac{1-S_{q+1}(u)}{1-S_q(u)}\right) = -\frac{1}{N-q} < 0$.

□

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